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Structural damage assessment in a cantilever beam with a breathing crack using higher order frequency response functions

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ABSTRACT

Vibration measurements offer an effective, inexpensive and fast means of non-destructive testing of structures and various engineering components. There are mainly two approaches to crack detection through vibration testing; open crack model with emphasis on changes in modal parameters and secondly, the breathing crack model focusing on nonlinear response characteristics. The open crack model based on linear response characteristics can identify the crack only at an advanced stage. Researchers have shown that a structure with a breathing crack behaves more like a nonlinear system, similar to that of a bilinear oscillator and the nonlinear response characteristics can very well be investigated to identify the presence of the crack. In the present study, the bilinear restoring force is approximated by a polynomial series and a nonlinear dynamic model of the cracked structure is developed using higher order frequency response functions. The effect of crack severity on the response harmonic amplitudes are investigated and a new procedure is suggested whereby the crack severity can be estimated through measurement of the first and second harmonic amplitudes.

1. Introduction

Structural health monitoring for damage assessment and residual life prediction has become an important research area in the recent past. Failures in structures and machine elements can be prevented through early detection of fatigue cracks using various non-destructive testing methods. Traditional non-destructive testing methods include dye-penetrant test, magnetic particle inspection, ultrasonic test etc, which have their own limitations and are often very expensive and inconclusive. Alternately, vibration based methods offer an effective, fast and convenient diagnostic tool for detection of the fatigue cracks in machine elements and structural systems. There are mainly two approaches to crack detection through vibration testing; open crack model with emphasis on changes in modal parameters and breathing crack model focusing on nonlinear response characteristics. In the first approach, crack is considered to be always open and it is modeled as a local flexibility [1] in the structure. Crack size and its location are investigated through changes in modal parameters such as natural frequency [2–5] and damping factor [6]. This approach, however, suffers from two major limitations. First, the changes in natural frequencies have been found to be significant only for a large crack size [7] and secondly, the measured shift in natural frequency cannot be conclusively attributed to crack alone, as it can also be affected by other factors such as wear, relaxation etc.

Recently, researchers have concentrated on nonlinear response characteristics of the cracked beam and have shown that the nonlinear behavior can be explained by the bilinear stiffness model of a breathing crack which opens and closes

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during a vibration cycle. Shen and Chu [8] formulated the bilinear equation of a simply supported beam using Galerkin procedure and obtained the response through numerical analysis. Chu and Shen [9] obtained a closed form solution for the bilinear oscillator under low frequency excitation and suggested that the changes in the dynamic behavior of cracked structures can be used to identify the size and location of the crack. Chati et al. [10] analysed the dynamics of a cracked beam as a piecewise linear system and obtained the bilinear frequency in terms of the two natural frequencies of the piecewise linear systems. Rivola and White [11] modeled the bilinear variation in the restoring force using Fourier series and employed bispectral analysis of response for the detection of a fatigue crack. The authors suggested that bicoherence, which is normalized version of the bispectrum, can be used as an indicator for the crack detection. Deviating from the bilinear model, Abraham and Brandon [12] considered the stiffness variation to be continuous and used Fourier series to simulate the varying restoring force. Sundermeyer and Weaver [13] examined the response of a cracked beam, forced at two frequencies with their difference equal to the resonant frequency of the system. Chondros and Dimarogonas [14], Tsyfansky et al. [15] have studied the nonlinear response characteristics of a cracked beam by continuous system modeling, whereas few other researchers [16–19] have modeled the effect as a local change in the stiffness matrix of a discrete system model. These research works mainly provide qualitative understanding of the nonlinear response characteristics of a cracked beam and very few have attempted quantitative assessment of the crack.

The present study explores the use of Volterra series response representation for developing a quantitative damage assessment technique for a cantilever beam with an edge crack. Volterra series [20,21] represents a nonlinear system through a set of first and higher order frequency response functions (FRFs). Several methods have been suggested [22,23] on parametric and non-parametric identification of polynomial form nonlinearities through measurement and analysis of these higher order FRFs. Rutolo et al. [24] have measured first and higher order FRFs from the response of a bilinear oscillator and observed that second- and fourth-order FRFs are more sensitive to the size of a crack than the first-order FRF. Very recently, Peng et al. have introduced the concept of nonlinear output frequency response function (NOFRF) and observed that the presence of a crack can be detected through investigation of various harmonic peaks in the NOFRF [25]. In the present work, the bilinear restoring force function of a cracked cantilever beam is approximated by a finite term polynomial series and the response amplitudes under harmonic excitation are obtained using Volterra series response representation. A procedure is then suggested for estimating the structural damage through measurement of the first and second harmonic amplitudes. The method is illustrated with numerical simulation for two different damage levels and it is found that the procedure provides an accurate estimation of the damage, even when the crack size is very small.

2. Volterra series response representation

Volterra series represents the input–output mapping of a physical system, with $f(t)$ as input excitation and $x(t)$ as output response, in a form of functional series given by

$$\begin{aligned} x(t) &= \int_{-\infty}^{\infty} h_1(\tau_1) f(t-\tau_1)\,d\tau_1 + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_2(\tau_1,\tau_2) f(t-\tau_1) f(t-\tau_2)\,d\tau_1\,d\tau_2 \\ &\quad + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_3(\tau_1,\tau_2,\tau_3) f(t-\tau_1) f(t-\tau_2) f(t-\tau_3)\,d\tau_1\,d\tau_2\,d\tau_3 + \cdots \\ &= x_1(t) + x_2(t) + \cdots + x_n(t) + \cdots \end{aligned} \tag{1}$$

with nth order response component, $x_n(t)$, expressed as

$$x_n(t) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} h_n(\tau_1,\ldots,\tau_n) f(t-\tau_1)\cdots f(t-\tau_n)\,d\tau_1\cdots d\tau_n \tag{2}$$

$h_1(\tau_1)$, is the familiar impulse response function of a linear system and $h_n(\tau_1,\ldots,\tau_n)$ are the nth order Volterra kernels. Higher order frequency response functions or Volterra kernel transforms can be defined as the multi-dimensional Fourier transforms of the higher order Volterra kernels as

$$H_n(\omega_1,\omega_2,\ldots,\omega_n) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_n(\tau_1,\tau_2,\ldots,\tau_n) \prod_{i=1}^{n} e^{-j\omega_i\tau_i}\,d\tau_1 d\tau_2\cdots d\tau_n \tag{3}$$

For a single-degree-of-freedom system, with polynomial form of stiffness nonlinearity, given by the equation of motion

$$m\ddot{x}(t) + c\dot{x}(t) + k_1 x(t) + k_2 x^2(t) + k_3 x^3(t) + \cdots = f(t) \tag{4}$$

with harmonic excitation

$$f(t) = A\cos\omega t = \frac{A}{2}e^{j\omega t} + \frac{A}{2}e^{-j\omega t} \tag{5}$$

the response, using Volterra series (1), is obtained as

$$x(t) = \sum_{n=1}^{\infty} \left(\frac{A}{2}\right)^n \sum_{p+q=n} {}^{n}C_q H_n^{p,q}(\omega) e^{j\omega_{p,q} t} \tag{6}$$

where $H_n^{p,q}(\omega)$ are the higher order FRFs given by

$$H_n^{p,q}(\omega) = H_n(\underbrace{\omega,\ldots,\omega}_{p\ \text{times}},\underbrace{-\omega,\ldots,-\omega}_{q\ \text{times}}) \qquad \omega_{p,q} = (p-q)\omega \tag{7}$$

For the polynomial form nonlinearity as given in Eq. (4), higher order FRFs can be synthesized from the lower order FRFs and the nonlinear parameters. After substituting Eq. (6) in the governing equation of motion (4) and equating the coefficients of $\left(\frac{A}{2}\right)^n e^{j\omega_{p,q}t}$, n=1, 2, 3..., one obtains

$$H_n^{p,q}(\omega) = -\frac{H_1(\omega_{p,q})}{{}^nC_q}\left[\begin{array}{l} k_2 \sum\limits_{\substack{p_i+q_i=n_i \\ n_1+n_2=n}} \{{}^{n_1}C_{q_1}H_{n_1}^{p_1,q_1}(\omega)\}*\{{}^{n_2}C_{q_2}H_{n_2}^{p_2,q_2}(\omega)\}+ \\ k_3 \sum\limits_{\substack{p_i+q_i=n_i \\ n_1+n_2+n_3=n}} \{{}^{n_1}C_{q_1}H_{n_1}^{p_1,q_1}(\omega)\}*\{{}^{n_2}C_{q_2}H_{n_2}^{p_2,q_2}(\omega)\}*\{{}^{n_3}C_{q_3}H_{n_3}^{p_3,q_3}(\omega)\} \end{array}\right] \quad \text{for } n>1 \tag{8}$$

2.1. Response harmonic amplitudes

For a harmonic excitation, response series given in (6) will consists of several harmonics and can be expressed as

$$x(t) = X_0 + |X(\omega)|\cos(\omega t+\phi_1) + |X(2\omega)|\cos(2\omega t+\phi_2) + |X(3\omega)|\cos(3\omega t+\phi_3) + \cdots \tag{9}$$

where the response harmonic amplitudes, $X(n\omega)$, are obtained as

$$X(n\omega) = \sum_{i=1}^{\infty} \sigma_i(n\omega) \quad \text{and} \quad \phi_n = \angle X(n\omega) \tag{10}$$

with

$$\sigma_i(n\omega) = 2\left(\frac{A}{2}\right)^{n+2i-2} {}^{n+2i-2}C_{i-1} H_{n+2i-2}^{n+i-1,i-1}(\omega) \tag{11}$$

Response amplitude for the first three harmonics, $\omega, 2\omega$ and 3ω, thus can be expressed as

$$X(\omega) = AH_1(\omega) + \frac{3A^3}{4}H_3(\omega,\omega,-\omega) + \text{higher order terms} \tag{12a}$$

$$X(2\omega) = \frac{A^2}{2}H_2(\omega,\omega) + \frac{A^4}{2}H_4(\omega,\omega,\omega,-\omega) + \text{higher order terms} \tag{12b}$$

$$X(3\omega) = \frac{A^3}{4}H_3(\omega,\omega,\omega) + \text{higher order terms} \tag{12c}$$

The higher order FRFs appearing in the harmonic amplitude series (Eqs. (12a–c)) can be synthesized from first-order FRFs and the nonlinear parameters, using Eq. (8), as

$$H_2(\omega,\omega) = -k_2H_1^2(\omega)H_1(2\omega) \tag{13a}$$

$$H_3(\omega,\omega,\omega) = H_1^3(\omega)H_1(3\omega)[2k_2^2H_1(2\omega)-k_3] \tag{13b}$$

$$H_3(\omega,\omega,-\omega) = H_1^3(\omega)H_1(-\omega)\left[\frac{2}{3}k_2^2\{H_1(2\omega)+2H_1(0)\}-k_3\right] \tag{13c}$$

$$\begin{aligned} H_4(\omega,\omega,\omega,-\omega) = -\frac{H_1(2\omega)}{4}\big[&k_2\{2H_1(-\omega)H_3(\omega,\omega,\omega)+6H_1(\omega)H_3(\omega,\omega,-\omega)+4H_2(\omega,\omega)H_2(\omega,-\omega)\} \\ &+k_3\{6H_1^2(\omega)H_2(\omega,-\omega)+6H_1(\omega)H_1(-\omega)H_2(\omega,\omega)\}\big] \end{aligned} \tag{13d}$$

Some important observations can be noted here about the response harmonics as under

(a) Odd harmonics are associated with odd order kernel transforms and even harmonics are associated with even order kernel transforms.
(b) Harmonic series, $X(n\omega)$, comprises of nth and higher order kernel transforms only.
(c) Second-order FRF, $H_2(\omega,\omega)$, is related only to the stiffness parameter, k_2, but third- and fourth-order FRFs are related to both the stiffness parameters, k_2 and k_3.

3. Bilinear modeling of the breathing crack in a cantilever beam

An ideal bilinear oscillator is a single-degree-of-freedom spring-mass-damper system (Fig. 1) with stiffness having two different values and can be represented by the equation of motion

$$m\ddot{x}(t)+c\dot{x}(t)+g[x(t)]=f(t) \tag{14}$$

where

$$g[x(t)]=\alpha k x(t)$$

with

$$\alpha=1 \quad \text{for } x(t)\prec 0 \quad \text{and} \quad \alpha\prec 1 \quad \text{for } x(t)\geq 0$$

A cantilever beam with a breathing edge crack (Fig. 2) also exhibits bilinear stiffness characteristics depending on whether the crack is open or closed. Using finite element method, the equation of motion can be expressed in terms of the mass, stiffness and damping matrices as

$$[M]\{\ddot{u}(t)\}+[C]\{\dot{u}(t)\}+[K]\{u(t)\}=\{F(t)\} \quad \text{when the crack is closed} \tag{15}$$

and

$$[M]\{\ddot{u}(t)\}+[C]\{\dot{u}(t)\}+[K-\Delta K]\{u(t)\}=\{F(t)\} \quad \text{when the crack is open} \tag{16}$$

The term ΔK in above equation represents the reduction in the stiffness matrix due to the crack and is a function of crack size as well as of crack location. Let ϕ_i be the mass normalized individual mode shapes of the eigen value problem corresponding to Eq. (15) such that the modal matrix can be expressed as

$$[\Phi]=[\phi_1 \;\; \phi_2 \;\; \cdots \;\; \phi_n]$$

Using the coordinate transformation $\{u(t)\}=[\Phi]\{q(t)\}$ and after pre-multiplying Eq. (15) and Eq. (16) by ϕ_1^T, one obtains the single-degree-of-freedom equation

$$\ddot{q}_1(t)+2\zeta_1\sqrt{\alpha}\omega_1\dot{q}_1(t)+\alpha\omega_1^2 q_1(t)=f_1(t) \tag{17}$$

where ζ_1 and ω_1 are respectively modal damping and natural frequency of the un-cracked beam in the first mode and

$$\alpha=\frac{\phi_1^T[K-\Delta K]\phi_1}{\omega_1^2}$$

when the crack is open and α=1, when the crack is closed.

Here damping matrix [C] has been assumed to be proportional and $q_1(t)$ represents the vibration displacement in the first mode. One can observe that Eq. (17) is similar to that of the bilinear oscillator given by Eq. (14). The forced vibration of the beam is generally dominated by the fundamental mode because of the frequency range of excitations and higher values

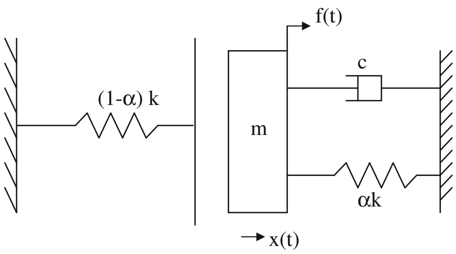

Fig. 1. Bilinear oscillator model of a spring mass damper system.

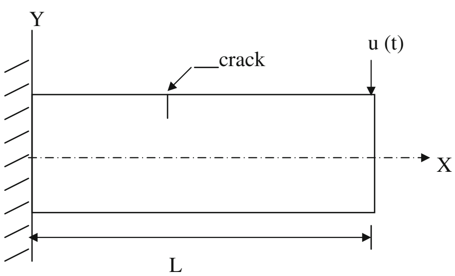

Fig. 2. A cantilever beam with an edge crack.

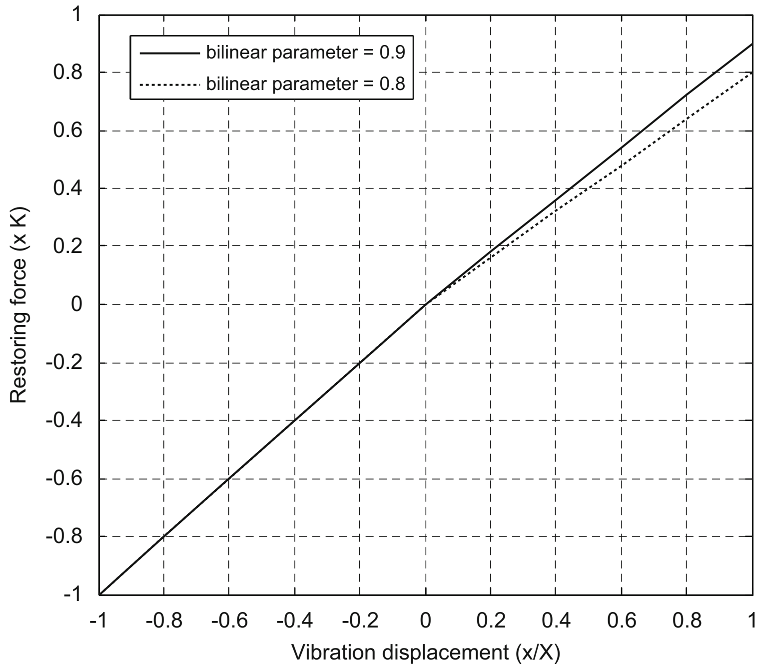

Fig. 3. Bilinear restoring force for α=0.9 and 0.8.

of modal damping in higher modes. Thus, forced vibration response of the cracked beam can very well be studied through the bilinear model as given in Eq. (14), where the parameter α represents the severity of the crack. As the crack depth increases, value of α decreases and the asymmetry in the restoring force increases (Fig. 3). In the following sections, a mathematical procedure based on Volterra series and higher order FRFs is presented for investigation of the spectral response characteristics, which is then employed for the crack severity assessment through measurement of first and second harmonic amplitudes in the response.

4. Harmonic amplitudes of the cracked beam through Volterra series

To obtain the response harmonic amplitudes through Volterra series, restoring force $g[x(t)]$ due to the bilinear stiffness of the cracked beam is approximated with a polynomial form $\hat{g}[x(t)]$ given as

$$\hat{g}[x(t)] = g_0 + k_1 x(t) + k_2 x^2(t) + k_3 x^3(t) + \cdots \tag{18}$$

Since the bilinear restoring force $g[x(t)]$ is a continuous function over the interval $(-X,X)$ of vibration displacement, it can be approximated to any desired accuracy with the polynomial $\hat{g}[x(t)]$ with suitable number of power terms in it (Weierstrass approximation theorem [26]). For the sake of simplicity here, the polynomial is considered up to the cubic power term and the coefficients, k_1, k_2, k_3 are computed by minimizing the error function E, given as

$$E(k_1, k_2, k_3) = \int_{-X}^{X} \{g[x(t)] - \hat{g}[x(t)]\}^2 \, dx \tag{19}$$

Applying $\partial E/\partial k_i = 0$ for $i = 1,2,3$ the coefficients are obtained as

$$k_1 = \frac{(1+\alpha)}{2}k, \quad k_2 = -\frac{15(1-\alpha)}{32X}k, \quad k_3 = 0 \tag{20}$$

From Eq. (12b), it can be seen that the harmonic amplitude, $X(2\omega)$, is associated with second-order kernel transform, $H_2(\omega,\omega)$, which is a function of the nonlinear parameter, k_2 (Eq. (13a)). The asymmetry in the bilinear restoring force leads to a non-zero value of k_2 in the polynomial approximation and explains the characteristic presence of the second harmonic in the response spectrum of a bilinear oscillator (Fig. 4a), where the non-dimensional frequency $r = \omega/\sqrt{k/m}$. On the other hand for system nonlinearity of symmetric form (i.e., $f(-x) = -f(x)$ as for a Duffing oscillator), only odd harmonics will be present in the response spectrum (Fig. 4b). Thus presence of even harmonics in the response spectrum indicates that the nonlinearity form is asymmetric as in the case of a bilinear oscillator. This forms the fundamental basis for detecting the presence of crack in a beam and other structural members. One has to note here that although the stiffness parameter k_3 happens to be zero for the bilinear function, third-order FRF, $H_3(\omega,\omega,\omega)$ will have non-zero value due to the parameter, k_2 and hence third harmonic will also be found in the response spectrum of the bilinear oscillator.

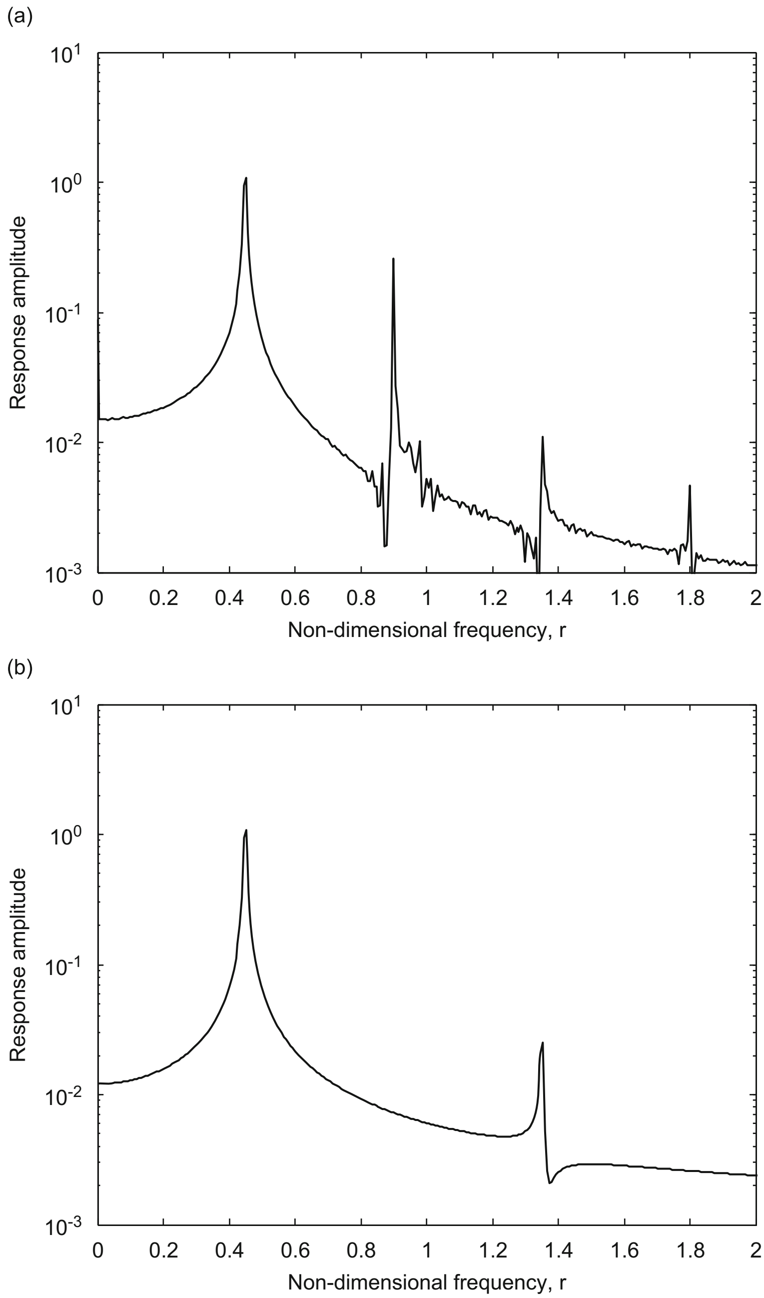

Fig. 4. (a) Response spectrum for bilinear oscillator with excitation frequency, $r=0.45$ and (b) response spectrum for Duffing oscillator with excitation frequency, $r=0.45$.

The polynomial approximation of the restoring force function of a bilinear oscillator, as given in Eq. (18), provides a basis to obtain the response harmonic amplitudes using higher order FRFs. The first and second harmonic response amplitude, following Eqs. (13a–d), then can be expressed as

$$X(\omega)=AH_1(\omega)+\frac{A^3}{2}k_2^2\{H_1(2\omega)+2H_1(0)\}H_1^3(\omega)H_1(-\omega)+\text{higher order terms} \tag{21a}$$

$$X(2\omega)=-\frac{A^2}{2}k_2H_1^2(\omega)H_1(2\omega)+\text{higher order terms} \tag{21b}$$

The response amplitudes of the first and second harmonics with excitation frequency varying over a range of $\omega/\sqrt{k/m}=0.1-1.3$ for two values of $\alpha=0.95$ and $\alpha=0.9$ have been obtained through numerical simulation and presented in Fig. 5a,b. It can be seen that the peak in first harmonic amplitude occurs slightly below $\omega/\sqrt{k/m}=1$. Actually the peak occurs at the bilinear natural frequency values $\omega_b/\sqrt{k/m}=0.987$ for α=0.95 and at $\omega_b/\sqrt{k/m}=0.974$ for α=0.9. Bilinear frequencies can be computed as [10,25]

$$\omega_b=\frac{2\omega_0\omega_1}{\omega_0+\omega_1}=\frac{2\sqrt{\alpha}}{1+\sqrt{\alpha}}\omega_0 \tag{22}$$

where ω_0 and ω_1 are respectively the fundamental natural frequency of the beam in un-cracked and cracked conditions.

Figs. 5a,b also indicate that the second harmonic amplitude is much lower than that of the first harmonic and this is an important point for practical signal measurement in presence of background and instrumental noise. The amplitude however is relatively higher in the neighborhood of $\omega=\omega_b/2$ and $\omega=\omega_b$. It is therefore recommended to use the excitation frequency in the vicinity of these frequencies to get measurable signal strength for the second harmonic in the response. It can also be observed from Figs. 5(a, b) that second harmonic amplitude increases as α decreases and there is no appreciable change in the first harmonic amplitude. This can be explained with the response harmonic series in Eq. (21b).

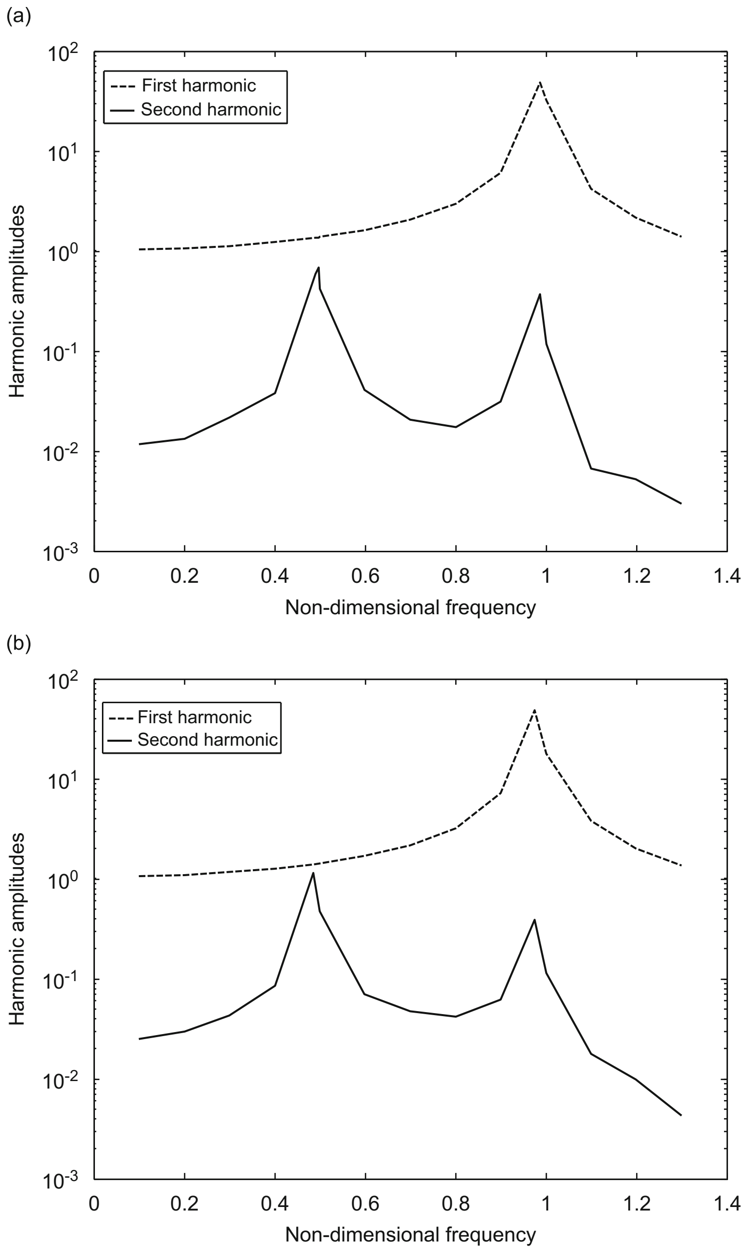

Fig. 5. (a) Response harmonic amplitudes for α=0.95 and (b) response harmonic amplitudes for α=0.9.

As α decreases for a growing crack, the nonlinear parameter k_2 increases (Eq. 20) and thus the harmonic amplitude $X(2\omega)$ which is directly related to k_2 (Eq. (21b)) also increases. The value of $1-\alpha$ can be used as an indicator of the crack severity and may be termed as crack severity index (CSI). Fig. 6 shows the variation in the spectral amplitudes of the first and second harmonic for different crack depth depicted by crack severity index varying between 0.01 to 0.10 for typical excitation frequencies $\omega/\sqrt{k/m}=0.3$ and 0.4. It can be seen that the second harmonic amplitude $X(2\omega)$ is much smaller than the first harmonic amplitude $X(\omega)$ for a small value of the index $1-\alpha$. In such cases an excitation frequency close to half of bilinear natural frequency, such as $\omega/\sqrt{k/m}=0.4$ will give higher signal strength $X(2\omega)$ than an excitation frequency $\omega/\sqrt{k/m}=0.3$. Fig. 7 shows the variation in spectral amplitude ratio $X(2\omega)/X(\omega)$ for different crack depth for the excitation frequencies. It is significant to note that the ratio is almost proportional to the severity index and thus can be potentially used as a measure of the damage. This aspect is further investigated through Volterra series expressions of the harmonic amplitudes in the following section.

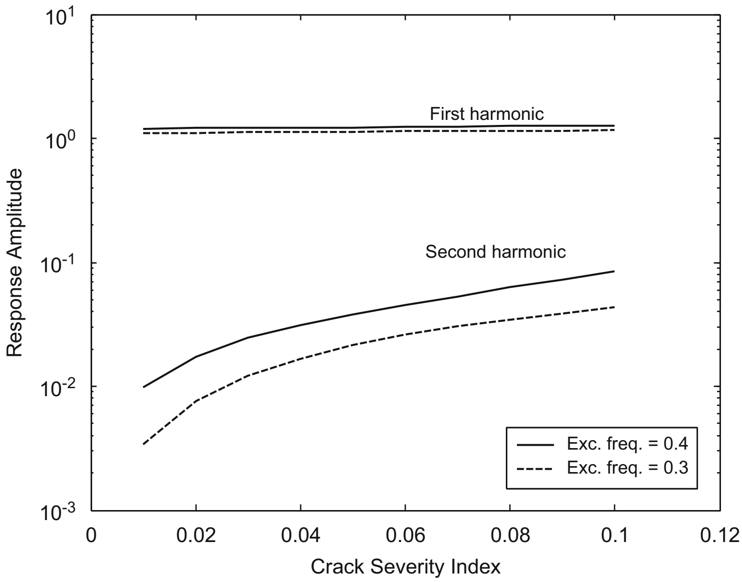

Fig. 6. Response harmonic amplitudes at different damage levels.

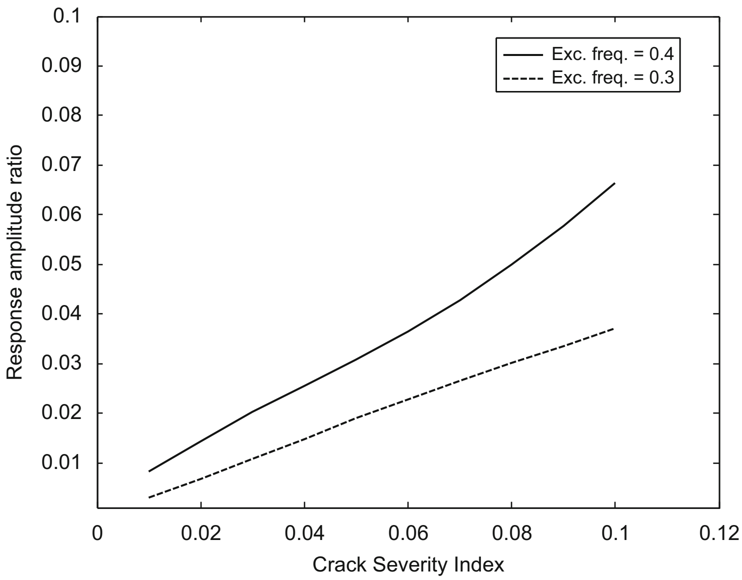

Figure. 7. Response amplitude ratio $X(2\omega)/X(\omega)$ at different damage levels.

5. Assessment of crack severity from response harmonic amplitudes

Successive terms of the harmonic amplitude series, given in Eqs. (21a,b), are in the ratio of k_2^2 and thus for weak bilinearity with small value of k_2, the harmonic amplitude series can very well be approximated by the first term only. Eqs. (21a,b) then become

$$X(\omega)=AH_1(\omega) \tag{23a}$$

and

$$X(2\omega)=-\frac{k_2}{2A}X^2(\omega)X*(2\omega) \tag{23b}$$

where $X*(2\omega)$ represents harmonic amplitude measured at frequency 2ω with excitation frequency also at 2ω. Using the relationship between k_2 and α in Eq. (20), one obtains

$$X(2\omega)=\frac{15(1-\alpha)k}{64A}X(\omega)X*(2\omega) \tag{24}$$

or

$$1-\alpha=\frac{64}{15}\frac{X(0)X(2\omega)}{X(\omega)X*(2\omega)} \tag{25}$$

where $X(0)$ is the static deflection which can be experimentally obtained with a static load. The value of the crack severity index (CSI), i.e., $1-\alpha$, can be easily estimated from experimental measurements using above Eq. (25). The procedure can be formulated in following steps.

Step 1: The excitation frequency ω and the excitation level A are appropriately selected such that second harmonic amplitude is distinctly measurable in the response. If the fundamental natural frequency of the system is known a-priori, one can select the excitation frequency close to half of it but not very close to the value. This also ensures that the vibration response will be mainly dominated by the first mode.

Step 2: The time history of response $x(t)$ is measured and the first and second harmonic amplitudes, $X(\omega)$ and $X(2\omega)$ are extracted from the response through Fourier series filtering. The system is then excited at twice the previous excitation frequency and first harmonic amplitude $X*(2\omega)$ is measured.

Step 3: The value $X(0)$ is obtained through a static loading test or can also be taken as $X(\omega)$ for a low value of excitation frequency ω. Finally the crack severity index is estimated using Eq. (25).

To investigate the effectiveness of the proposed damage assessment technique, a numerical simulation is carried out for the equation of motion (14) with damping ratio taken as 0.02. Excitation frequency is selected at $\omega/\sqrt{k/m}=0.3$ (Case I) and $\omega/\sqrt{k/m}=0.4$ (Case II) and the crack severity index is varied between 0.01 and 0.10. Fig. 8 shows the accuracy of estimation of the crack severity index for the two cases. The estimates are more accurate for smaller crack sizes (α close to 1). However, errors for larger crack sizes are higher (6.7% for Case I and 12.4% for Case II at $1-\alpha$=0.1). The errors

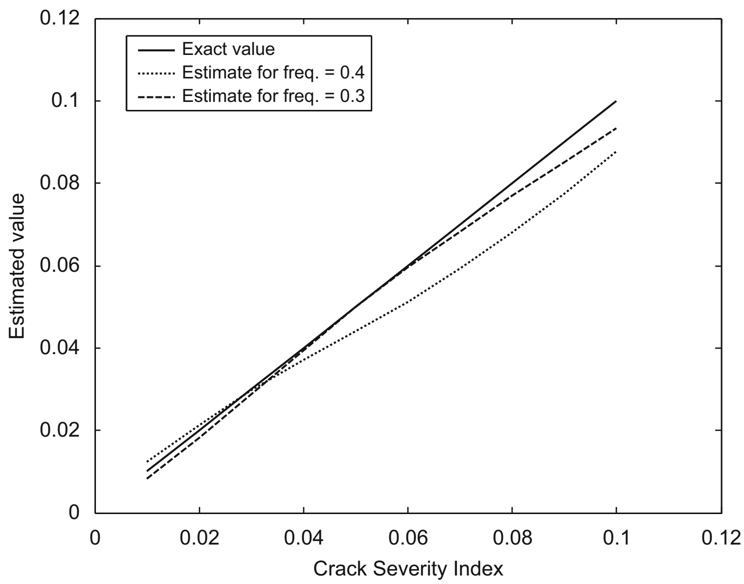

Fig. 8. Estimation of crack severity with $\omega/\sqrt{k/m}=0.3$ and 0.4.

can be attributed to the truncation of polynomial series representation of the bilinear restoring force in Eq. (18) and to first term approximation of the response series given in Eqs. (21a,b). Case I, with excitation frequency relatively away from half the bilinear natural frequency, gives more accurate estimation as the truncation error from higher order terms in Eqs. (21a,b) decreases due to lower values of $H_1(\omega)$ and $H_1(2\omega)$. However, when the crack size is small, response amplitude $X(2\omega)$ may not be distinctly measurable if the excitation frequency is selected far away.

6. Conclusion

The nonlinear response of a cracked beam is analysed using Volterra series response representation. The bilinear restoring force due to crack open and crack closed modes is approximated by a polynomial series and the first- and second-order frequency response functions are developed in terms of the bilinear parameter. The effect of crack severity on the response harmonic amplitudes are investigated and a new technique is suggested whereby the crack severity can be estimated through measurement of the first and second harmonic amplitudes. The estimate is found to be more accurate for smaller crack size. The procedure also discusses proper selection of the excitation level and excitation frequency for the estimation.

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